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Brane-antibrane as a defect of tachyon condensation

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Abstract: In a tachyon effective field theory of a non-BPS brane, we construct a classical solution representing a parallel brane-antibrane. The solution is made of a kink and an antikink placed at antipodal points of S^1. Estimation of the brane energy suggests an excitation of a string connecting the two branes, even though the theory is abelian. We performed fluctuation analysis around the obtained solution, and find the structure of the supersymmetry breaking by the co-existence of the brane and the antibrane. We discuss possible processes of the pair-annihilation of the brane defect.

Keywords: D-branes, Tachyon Condensation, Supersymmetry Breaking.

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1. Introduction and setup

After the success of K-theory in string theory, a standpoint that various D-brane configurations can be obtained by the open string tachyon condensation is established. This viewpoint is the third one to provide the description of D-branes, as well as the conformal field theory approach and the supergravity description. Around this new possibility, there are two currents of study, one is the verification of Sen's conjecture [1]–[3] on tachyon condensation with use of string field theories, and another is to utilize this new description to describe off-shell dynamics of D-branes including time-dependent backgrounds and application to realistic braneworld models.

Our attempt in this note is oriented rather to the second point above. The aim of this note is to see how far we can proceed for constructing brane configurations as a defect of the tachyon condensation in string theory,[1] by restricting our attention to the brane-antibrane configurations. It is known that, to obtain a desired brane configuration as a tachyon condensate, one has to prepare an appropriate number of non-BPS branes from the first place. For example, if one wants to construct a solution of two parallel D-branes, one has to prepare at least two unstable branes of higher dimensional worldvolume. Therefore the

[1] The early studies of open string tachyon dynamics, see [4].

inter-brane interactions of the resultant defect branes are already encoded in the original non-abelian setup. Then one can extract from the solution the off-diagonal interaction which corresponds to the excitation of the string connecting two branes, and it was found in [5] that this excitation reproduces the lowest mass squared of the string excitation.[2]

However, we will show explicitly that a solution representing brane and antibrane is possible in an abelian setup. Since this brane-antibrane should be a kink-antikink (or rather to say, wall-antiwall [7]), this can be constructed by a simple abelian model with only one tachyon field. The intriguing aspect here is that non-abelian structure is expected to emerge from the abelian model.

Another motivation for the construction of the brane-antibrane solution in superstring tachyon models is to realize in string theory the outcome of the field theoretical analysis in non-linear sigma models,[3] phenomenology with the extra dimensions, and braneworlds. In particular, brane-antibrane system breaks supersymmetries, which is phenomenologically preferable. We perform the fluctuation analysis of the constructed solution, and investigate how the spectra with supersymmetry breaking appears in the superstring tachyon model. In accordance to the recent development on the tachyon cosmology, we also study the time-dependent decay process of the brane and antibrane.

For concreteness we shall use the two-derivative truncation of the boundary string field theory (BSFT) action [10, 11] for the field theory of the tachyon on the non-BPS D-brane. Section 2 provides a brane-antibrane solution of this theory, and in section 3 the force between the branes is estimated to suggest the off-diagonal string interaction. In section 4 we study the fluctuation spectra and find supersymmetry breaking structure. Section 5 gives the decay width of the brane and antibrane. Conclusion and discussions are provided in the final section 6. In appendix we find similar solutions exist also in the BSFT with higher derivative corrections.

2. Brane-antibrane solution

2.1 BSFT action and its solutions

As mentioned in the introduction, we use the two-derivative truncation of the BSFT action of a non-BPS brane [11], some of which are called Minahan-Zwiebach (MZ) model [12, 13]. This kind of BSFT model exhibits very interesting properties which are consistent with string theory. In particular, exact classical solutions representing a D-brane exist, and they have appropriate Ramond-Ramond (RR) charges. Furthermore, the fluctuation spectra around the soliton solutions are exactly solvable, and found to be consistent with D-brane spectra [12]–[14].

[2] Another interesting example is non-commutative solitons in an abelian worldvolume gauge theory of D-branes [6]. In this case because of the non-commutativity the worldvolume theory effectively becomes non-abelian $U(\infty)$, and off-diagonal modes in the sectors of the solitons (lower-dimensional D-branes bounded) reproduce the spectra of strings stretched between them.

[3] An interesting example was provided in [8], where the kinky-lump solution in a non-linear sigma model constructed in [9] was realized as a D-brane configuration made of superstring tachyon condensation.

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The MZ model action for a single non-BPS D$(p+1)$-brane is given by

$$S = -\int d^{p+2}x\, (\mathcal{K}')^2 \,(1 + \partial_\mu T \partial^\mu T)\,, \tag{2.1}$$

where $\mathcal{K}'(T)$ is a function of T which specifies the tachyon potential. For the action obtained by the derivative truncation of the BSFT, we have

$$\mathcal{K}' = \exp\left(-\frac{T^2}{2}\right). \tag{2.2}$$

In the action (2.1), for simplicity we ignored overall factor of the non-BPS brane tension. In section 2 and 3, we neglect the dependence on the coordinate other than one world volume direction x. The dimension of the tachyon field is appropriately normalized with α'.

A characteristic point of this action is that the potential term $(\mathcal{K}')^2$ is multiplied also on the kinetic term. For the BSFT potential $(\mathcal{K}')^2 = \mathrm{e}^{-T^2}$, a perturvative vacuum $T=0$ is meta-stable, and there are two true vacua, $T = \pm\infty$. The equations of motion are given by

$$\partial_\mu \partial^\mu T - \frac{\mathcal{K}''}{\mathcal{K}'}\,(1 - \partial_\mu T \partial^\mu T) = 0\,, \tag{2.3}$$

provided that $\mathcal{K}' \neq 0$. It is known that the above Lagrangian allows a very simple linear solution

$$T = x - x_0 \tag{2.4}$$

which represents a Dp-brane, since according to Sen's conjecture the kink solution which interpolates two vacua ($T = \pm\infty$) is a BPS D-brane with codimension-one world volume.

The location of the D-brane is determined by the equation $T(x) = 0$, since the point $T = 0$ is just the point between the two true vacua, and any kink solution should pass this mid point. Therefore, the above single kink solution represents a BPS Dp-brane located at $x = x_0$. It should be noted that another solution $T = -(x - x_0)$ denotes an anti-Dp-brane, since the coupling to the RR gauge field $\int \mathrm{e}^{-T^2} dT \wedge C$ gives negative contribution. Therefore, if the solution passes zero of T from negative to positive it represents a BPS D-brane while if it passes in the opposite way it is an antibrane.

2.2 Brane-antibrane solutions

Let us proceed to obtain new solutions representing a pair of a brane and an antibrane. What we require for a solution of the brane and antibrane is that it satisfies the following properties:

- At both of the spatial infinities $x = \pm\infty$, T should be located at near one of the true vacuum : $T = \infty$.
- The equation $T(x) = 0$ with the solution T should have two solutions for x, which are the locations of the brane and the antibrane. $\partial_x T$ has different signs at these zeros of the solution T.

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Let us find a solution satisfying these properties. In fact, it is possible to construct a general solution for the equations of motion (2.3) if we restrict our attention to a field configuration which depends only one direction x. By multiplying $\partial_x T$ on (2.3), the equation of motion is recast into the form

$$\frac{\mathcal{K}''}{\mathcal{K}'}\partial_x T = \frac{\partial_x T}{1-(\partial_x T)^2}\partial_x^2 T \,. \tag{2.5}$$

This can be integrated by x, and the result is

$$-\log \mathcal{K}' = \frac{1}{2}\log\left(1-(\partial_x T)^2\right) + \text{const.} \tag{2.6}$$

Thus we obtain a relation

$$(\partial_x T)^2 = 1 - A(\mathcal{K}')^{-2} \,, \tag{2.7}$$

where A is an integration constant parameter. One observes that the vanishing A gives the simple linear solution (2.4). Therefore this parameter A is the one which characterizes the deviation from the BPS single kink solution toward the brane-antibrane solution. The physical parameter of the brane and antibrane which is expected is the inter-brane separation, and we will relate this to the above A. Before going there, the following boundary condition will be found to be useful:

$$T = T_0 \,, \qquad \partial_x T = 0 \qquad \text{at} \qquad x = x_0 \,. \tag{2.8}$$

Then, for the case of BSFT potential (2.2), we have $A = \mathrm{e}^{-T_0^2}$, and integration of the equation (2.7) gives an explicit expression for the general solution:

$$x = x_0 \pm \mathrm{e}^{T_0^2/2}\int_{T_0}^{T}\frac{1}{\sqrt{\mathrm{e}^{T_0^2}-\mathrm{e}^{T^2}}}\,dT \,. \tag{2.9}$$

The BPS limit is $T_0 \to \infty$, though the above expression becomes meaningless in that limit. We will find later that this is because the brane and the antibrane are pushed out to the spatial infinities in this limit.

2.3 Properties of the solution

Here we shall show that the solution (2.9) is actually a solution representing a brane-antibrane pair. In the following of this section and the next section, we consider only the BSFT potential (2.2) for simplicity. Let us suppose that $T_0 < 0$ and $x_0 = 0$. This does not lose any generality since the original system is invariant under $T \to -T$ and also translationally invariant.

Interestingly, the solution (2.9) can be consistent only if $|T| < |T_0|$. So let us study what happens around the critical point $T \sim \pm T_0$. For the tachyon field near the critical point the integrand in the solution (2.9) seems to be dangerous because of a possible divergence. In order to check the behavior around the critical point, we expand the tachyon as

$$T = T_0 + \delta \,, \tag{2.10}$$

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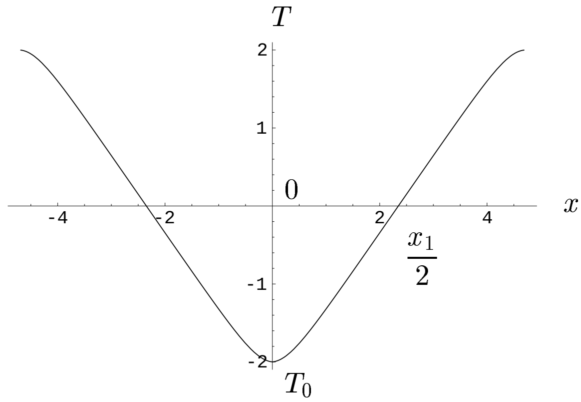

Figure 1: Solution (2.9) with $x_0 = 0$ and $T_0 = -2$.

where positive δ is taken to be infinitesimally small. Then the integral is estimated as

$$x_1 - x = \int_{T_0-\delta}^{T_0} \frac{dT}{\sqrt{1 - e^{T^2 - T_0^2}}} \sim \int_0^{\delta} \frac{1}{\sqrt{2|T_0|\delta'}} d\delta' = \sqrt{\frac{2\delta}{|T_0|}} . \tag{2.11}$$

Thus the integral is finite and has no divergence. This means that if we plot the solution in the x-T space the solution is a curved line with a finite length, connecting $(x = 0, T = T_0)$ and $(x = x_1, T = -T_0)$. In fact, the second term in (2.9) is convergent for $T = -T_0$, thus there is the end of the base space at $x = \pm x_1$ where

$$x_1 \equiv e^{T_0^2/2} \int_{T_0}^{-T_0} \frac{1}{\sqrt{e^{T_0^2} - e^{T^2}}} dT . \tag{2.12}$$

How should this finiteness be understood ? One idea to interpret this strange property is to assume that the solution lives in a compact space, $-x_1 \le x < x_1$. If one assumes this, the size of the compactified dimension $(2x_1)$ determines the parameter T_0 through the above equation. The other idea is that one can have a periodic solution, ends up with an infinite repetition of the solution. Here the period is of course the same as the first case, $2x_1$.

It should be noted that the solution $T(x)$ has two zeros in the definition region $-x_1 \le x < x_1$. This is almost obvious since substituting $T = 0$ in (2.9) gives $x = \pm x_1/2$, because the integrand is symmetric under $T \to -T$. The zeros of the tachyon profile represent the location of the D-branes. The tachyon potential e^{-T^2} has two vacua, $T = \pm\infty$, so there are two types of topological defects: a kink and an anti-kink, with regard to its orientation. In our case, $\partial_x T(x_1/2)$ is positive while $\partial_x T(-x_1/2)$ is negative, hence our solution (2.9) is actually a pair of a kink and an antikink. Therefore, we conclude that this solution represents the brane-antibrane pair. The inter-brane separation is x_1.

We plot a numerical solution in figure 1. One can easily see that most part of the solution is linear, and the edge of the lines (at $x = 0, \pm x_1$) are smoothly connected. The kink and the antikink are joined.

For the later purpose, we estimate the brane separation in terms of T_0. Suppose that we have very large $|T_0|$. Then the integrand in the solution (2.9) is approximated by a constant in almost all the region of T. This constant is actually given by the value at

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$T=0$, as

$$\left.\frac{\partial T}{\partial x}\right|_{T=0} = \pm q\,, \tag{2.13}$$

where $q \equiv \sqrt{1-\mathrm{e}^{-T_0^2}}$. Therefore, we can estimate that in this large $|T_0|$ limit the brane separation x_1 can be given by

$$x_1 \equiv \mathrm{e}^{T_0^2/2}\int_{T_0}^{-T_0} \frac{1}{\sqrt{\mathrm{e}^{T_0^2}-\mathrm{e}^{T^2}}}\,dT \sim \int_{T_0}^{-T_0}\frac{1}{q}\,dT = \frac{2|T_0|}{q}. \tag{2.14}$$

Therefore, approximately the brane separation x_1 is given by $2|T_0| + \mathcal{O}(\mathrm{e}^{-T_0^2})$.

Summing up the above estimations, an approximate form of the solution for positive x is given as

$$T \sim \begin{cases} -|T_0| + \dfrac{|T_0|}{2}x^2 & 0 < x < \dfrac{q}{|T_0|} \\ q\left(x - \dfrac{x_1}{2}\right) & \dfrac{q}{|T_0|} < x < \left(x_1 - \dfrac{q}{|T_0|}\right) \\ |T_0| - \dfrac{|T_0|}{2}(x-x_1)^2 & \left(x_1 - \dfrac{q}{|T_0|}\right) < x < x_1\,. \end{cases} \tag{2.15}$$

3. Force between the branes

It is obvious that since this solution is a wall antiwall pair, the energy is reduced if the distance between the two approaches zero while the compactification radius fixed. However, these two parameters are related for the obtained solution, of course. The obtained configuration is meta-stable because the brane and the antibrane are located in an antipodal manner. The instability associated with the off-shell deformation of decreasing the inter-brane distance is the tachyonic instability which comes from the interaction between the two.

In string theory, naively this interaction mode is a closed string mode such as gravity and RR-interaction which are attractive force between the brane and the antibrane. However, from the first setup we neglected the closed string channel, so the appearance of the gravity interaction is hardly expected here.

Let us recall that the gravity and the RR interaction are arranged into the one loop amplitude of open string stretched between the two, if one sums up all the massive modes of closed strings. This is the s-t channel duality. Now in the dual channel (open string) description, we prepared only the lowest modes (tachyons), that is why we cannot expect the reproduction of the gravity interaction. Then, what is the interaction of branes in our case? Instead of the closed string interaction, we may expect only the lowest interaction of stretched open string. This is a massive state of the mass $\sim x_1$.

The above argument shows that the inter-brane interaction is now dictated by the massive state with the mass $\sim x_1$, not by the gravity interaction. This should be seen in the solution constructed in this paper. For that purpose, let us evaluate the potential between the branes. Denoting the energy of the original single brane $T=x$ as E_0,

$$E_0 \equiv \int_{-\infty}^{\infty} dx\,\left[\mathrm{e}^{-T^2}\left(1+(\partial_x T)^2\right)\right]_{T=x} = 2\int_{-\infty}^{\infty} dx\,\mathrm{e}^{-x^2}\,, \tag{3.1}$$

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we can evaluate the potential energy by an excess energy ΔE of the brane-antibrane configuration compared to twice of the energy E_0 of the single isolated brane,

$$\Delta E = \int_{-x_1}^{x_1} dx\, \mathrm{e}^{-T^2} \left(1 + (\partial_x T)^2\right) - 2E_0 \,. \tag{3.2}$$

The multiplication factor 2 in the last term is necessary since we have two branes.

By using (2.9) with $x_0 = 0$, we change integration variable from x to T and obtain

$$\begin{aligned} \Delta E &= 2\int_{T_0}^{-T_0} dT\, \frac{2\mathrm{e}^{-T^2} - \mathrm{e}^{-T_0^2}}{\sqrt{1-\mathrm{e}^{T^2-T_0^2}}} - 4\int_{-\infty}^{\infty} dT\, \mathrm{e}^{-T^2} \qquad (3.3)\\ &= 8\left[\int_0^{|T_0|} dT\, \mathrm{e}^{-T^2} \left(\frac{1}{\sqrt{1-\mathrm{e}^{T^2-T_0^2}}} - 1\right) - \int_{|T_0|}^{\infty} dT \mathrm{e}^{-T^2} - \int_0^{|T_0|} dT\, \frac{\mathrm{e}^{-T_0^2}}{2\sqrt{1-\mathrm{e}^{T^2-T_0^2}}}\right]. \end{aligned}$$

We can see that each term in the last line gives contributions of order $\mathcal{O}(\mathrm{e}^{-T_0^2})$ with possible powers of T_0. Since the brane-antibrane distance is $x_1 \sim 2|T_0|$, we obtain

$$\Delta E \sim -\frac{1}{x_1^n}\mathrm{e}^{-x_1^2/4} + \text{higher orders}\,. \tag{3.4}$$

Here we have not determined the precise value of the integer n (this can be negative), since the important part is the exponential. If we regard this as a Yukawa-type potential

$$V(x_1) = \frac{e_1 e_2}{x_1^n}\mathrm{e}^{-Mx_1}\,, \tag{3.5}$$

where M is the typical mass of the particle which causes the interaction, we see

$$M \sim x_1\,, \tag{3.6}$$

up to a normalization.[4] Here we see the appearance of a new mass scale $\sim x_1$. Since the interbrane interaction should come from strings connecting the branes, this suggests that the typical mass scale for these strings should be of order $\sim x_1$. This is consistent with the string theory mass spectra. However, the string connecting the two propagates along the branes while the particle producing the potential (3.5) propagates transversely. Although this difference might be explained by the duality, precise reproduction of the corresponding string excitation is yet to be found.

4. Fluctuation analysis

In this section we investigate the low energy effective field theories induced on the brane and antibrane. We do not consider the effective theories on each brane separately, since there exists weak interactions between the branes, as we have seen in the previous section. Because of this weak interaction, the degenerate mass spectra on each brane resolves to

[4] The above argument does not completely ensure that the mass is actually linear in x_1, since the mass itself is expected to depend on the separation x_1 and in principle one cannot extract the mass dependence only from the excess energy.

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form a fine structure. This resolution is actually due to the supersymmetry breaking, since a single brane (a single wall) itself is supersymmetric and is a BPS state while the coexistence of the brane and the antibrane breaks the supersymmetry. Since the inter-brane interaction as well as the supersymmetry breaking are exponentially suppressed as a function of the distance between the brane and antibrane, the resulting tower of boson and fermion spectra exhibits fine splittings.

4.1 Tachyon fluctuation

First let us derive the lagrangian for the fluctuation of the tachyon fields around the classical solution $T_{\rm cl}$ (2.9). We expand the field as

$$T(x, y^{\hat{\mu}}) = T_{\rm cl}(x) + t(x, y^{\hat{\mu}})\,, \tag{4.1}$$

where y denotes the defect brane worldvolume coordinate, $\hat{\mu} = 0, 1, \ldots, p-1$. Substituting this expression to the original lagrangian (2.1) and pick up the terms quadratic in t, we obtain

$$\mathcal{L}_t = -\left(\mathcal{K}'\right)^2 \left[\left(\left(\frac{\mathcal{K}''}{\mathcal{K}'}\right)^2 + \frac{\mathcal{K}'''}{\mathcal{K}'}\right)(1 + (\partial_x T_{\rm cl})^2)t^2 + \frac{\mathcal{K}''}{\mathcal{K}'} 4\partial_x T_{\rm cl} t \partial_x t + (\partial_x t)^2 + (\partial_{\hat{\mu}} t)^2\right]. \tag{4.2}$$

Here and in the following, $\mathcal{K}$ is the function into which the classical solution $T_{\rm cl}$ is substituted. Following the technique developed in [12, 13], we make a field redefinition as

$$\hat{t} \equiv \mathcal{K}'(T_{\rm cl})\, t\,. \tag{4.3}$$

Then after a partial integration we obtain

$$\mathcal{L}_t = -\hat{t}\left[\left(-\partial_x^2 + V_{\rm tachyon}(T_{\rm cl})\right) - \partial_{\hat{\mu}}^2\right]\hat{t}\,, \qquad V_{\rm tachyon}(T_{\rm cl}) \equiv \frac{\mathcal{K}'''}{\mathcal{K}'}\,. \tag{4.4}$$

Therefore, if we expand the fluctuation fields as

$$t = \sum_n t_n(y^{\hat{\mu}}) u_n(x) \tag{4.5}$$

in which the function $u_n(x)$ solves the eigenvalue equations

$$\left(-\partial_x^2 + V_{\rm tachyon}(T_{\rm cl})\right) u_n(x) = m_n^2 u_n(x), \tag{4.6}$$

the corresponding mode $\hat{t}_n$ has the mass m_n,

$$\partial_{\hat{\mu}}^2 \hat{t}_n(y^{\hat{\mu}}) = m_n^2 \hat{t}_n(y^{\hat{\mu}})\,. \tag{4.7}$$

For the BSFT potential (2.2), $V_{\rm tachyon}(T_{\rm cl})$ is given by ${T_{\rm cl}}^2 - 1$. Therefore if one has a single D-brane solution as a background, the potential becomes harmonic and the Schrödinger equation (4.6) becomes solvable, results in a spectrum with equal spacing.

Since we have two branes, there should be two almost massless modes as the low energy modes. Among them, we may expect that there is a massless Nambu-Goldstone

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(NG) boson coming from the breaking of the translational symmetry of the original system. The other modes should be tachyonic, since this scalar mode should be related to the inter-brane interaction which was shown to be a negative mode in the previous section. Let us see this in more detail using the BSFT potential (2.2).

In fact, the potential which fluctuation modes feel is the double-well type. The potential $V(T_{\rm cl})$ has two minima at the zeros of the solution $T_{\rm cl}$, $x = \pm x_1/2$. If the inter-brane separation is large enough, we may expect massless NG modes for each brane. Now, if the separation is not so large, these two NG modes are not really a NG mode, and they interact with each other. A basic knowledge on the quantum mechanics shows that the symmetric wave function is the true lowest mode while the antisymmetric wave function is the first excited level. Thus we expect that this symmetric one is tachyonic and the antisymmetric one is the massless NG mode.

The correct expression for the massless NG mode is well-known,

$$t(x) = \partial_x T_{\rm cl}\,. \tag{4.8}$$

It is easy to check that this satisfies (4.6) with $m^2 = 0$, if we use the equations of motion for $T_{\rm cl}$, (2.3). And one can check that this NG mode is actually antisymmetric under the parity transformation $x \leftrightarrow -x$. The expression (2.9) of the solution shows $T_{\rm cl}$ is an even function of x, thus its derivative $\partial_x T_{\rm cl}$ is an odd function (antisymmetric with respect to the double well potential). Figure 2 is the plot of the NG mode.

One observes that the NG mode is almost flat, as is obvious from the expression (4.8) and the fact that the solution (2.9) is almost linear around the bottom of the potential. In fact, the single kink solution is a simple linear function $T_{\rm cl} = x$, thus its NG mode is just constant. To describe in a different manner, we see that the potential for the fluctuation (4.6) is that of a harmonic oscillator for $T_{\rm cl} = x$, thus the lowest mode is gaussian, $\hat{t} = A\exp(-x^2)$. This shows that through the field redefinition (4.3) the fluctuation t is constant.

The massless NG mode is an antisymmetric combination of the flat configuration, thus we expect that the tachyonic one is a symmetric combination, that is, almost constant everywhere. We don't know the exact expression for that, however we can obtain an upper bound for its mass squared. First, from the potential $V_{\rm tachyon}$, one can see that its mass squared should be greater than -1. Second, since we know that it is the lowest mode of the Schrödinger equation (4.6), we can use the following inequality for any smooth function f (with appropriate boundary conditions)

$$m^2 \le \frac{\int dx\, f\left(-\partial_x^2 + V_{\rm tachyon}(T_{\rm cl})\right) f}{\int dx\, f^2}. \tag{4.9}$$

From the observation above, we know that the true fluctuation is almost a constant function everywhere, thus a constant function is actually a good test function. Substituting this with noting that one has to perform the field redefinition (4.3), one obtains

$$m^2 \le \frac{\int dx\, {\rm e}^{-{T_{\rm cl}}^2/2}\left(-\partial_x^2 + V_{\rm tachyon}(T_{\rm cl})\right){\rm e}^{-{T_{\rm cl}}^2/2}}{\int dx\, {\rm e}^{-{T_{\rm cl}}^2}}\,. \tag{4.10}$$

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Using the equations of motion, the numerator of the right hand side reads

$$\int dx\, \mathrm{e}^{-T_{\rm cl}{}^2} \left((\partial_x T_{\rm cl})^2 - 1\right) = -2x_1 \mathrm{e}^{-T_0^2}, \tag{4.11}$$

where at the last equality we have used the relation (2.7). Since this is negative, it is shown that the system admits a tachyonic fluctuation, as expected. This result can be derived without referring to the special form of $\mathcal{K}'$ (2.2). Only if the parameter A in (2.7) is non-zero (not the BPS limit), the above estimation gives a tachyonic fluctuation.

The constant test function is a better approximation for a larger separation of the branes. Therefore, the mass squared of this tachyonic fluctuation can be estimated in the large brane separation as

$$m^2 \sim -\frac{2}{\sqrt{\pi}} |T_0| \mathrm{e}^{-T_0^2}\,. \tag{4.12}$$

For a very large separation of the branes, we can see that this state is nearly massless. If we take the BPS limit ($|T_0| \to \infty$), the above state becomes exactly massless, which is consistent with the fact that this limit is actually separating the brane and the antibrane to the opposite spatial infinities and turning off the inter-brane interactions to reproduce two sets of the spectra of a single BPS brane. We will show later in section 4.4 that the mass squared of tachyon (4.12) can also be obtained by means of the low-energy theorem associated with the supersymmetry breaking.

In terms of string theory, the possible interpretation of the tachyonic state with the mass squared (4.12) is given as follows. The matter content of the coincident brane-antibrane system consists of two transverse scalar fields $X^{(1)}$ and $X^{(2)}$, two gauge fields and a comlex tachyon field φ. Here we assumed for simplicity a single transverse direction. The tachyon field φ becomes massive if the brane separation is larger than the string scale. Note that this massive tachyon field is not directly related with the tachyon fields which we have discussed, T and t. There is the following coupling between the scalar fields X and the massive tachyon φ [25],

$$(X^{(1)} - X^{(2)})^2 |\varphi|^2 \exp(-|\varphi|^2)\,. \tag{4.13}$$

Since we are looking at the low energy excitation, the massive φ may be integrated out, then the above coupling gives a mass term for the linear combination $X^{(1)} - X^{(2)}$. For the large separation, the mass of φ becomes large, and the integration gives only small contribution, thus the mass generated from this term is expected to be very small. This is expected to give the expression (4.12), since in the above coupling there appears the exponential, and the typical mass scale of φ is $\sim x_1 \sim T_0$. On the other hand, another linear combination $X^{(1)}+X^{(2)}$ has no coupling like above, therefore the mass is not generated by the integration and remains massless. This remaining massless mode is identified with the NG mode in our fluctuation analysis, the antisymmetric combination of the flat configurations for t.

4.2 Fermion fluctuation

It is known that on the non-BPS branes there are fermions which couple to the tachyon.

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The leading order fermion terms are given by [13]

$$S = -\int d^{10}x\ \left(\mathcal{K}'(T)\right)^2 \left[\frac{i}{2}\bar{\psi}\Gamma^\mu\partial_\mu\psi + W(T)\bar{\psi}\psi\right], \tag{4.14}$$

where the Yukawa coupling is supplied with

$$W(T) \equiv -\frac{\mathcal{K}''(T)}{\mathcal{K}'(T)}. \tag{4.15}$$

In this subsection we adopt $p = 8$ for definiteness. For the fermion spectrum in the tachyon solution background, we follow the analysis [13]. Performing the field redefinition

$$\chi \equiv \mathcal{K}'(T_{\rm cl})\psi \tag{4.16}$$

and decomposing the fields and the gamma matrices as

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \qquad \Gamma^x = \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix}, \qquad \Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix} \qquad (\mu = 0, \ldots, 8), \tag{4.17}$$

we have the Schrödinger equation for $\chi_\pm \equiv \chi_1 \pm \chi_2$ as

$$\left(-\partial_x^2 + V^{(\pm)}_{\rm fermion}(T_{\rm cl})\right)\chi_\pm = m_\pm^2\chi_\pm, \tag{4.18}$$

where the potential is given as

$$V^{(\pm)}_{\rm fermion} = (W(T_{\rm cl}))^2 \pm \partial_x W(T_{\rm cl}) = \left(\frac{\mathcal{K}''}{\mathcal{K}'}\right)^2 \mp \left[\frac{\mathcal{K}'''}{\mathcal{K}'} - \left(\frac{\mathcal{K}''}{\mathcal{K}'}\right)^2\right]\partial_x T_{\rm cl}. \tag{4.19}$$

For the BSFT potential (2.2), $V^{(\pm)}_{\rm fermion} = {T_{\rm cl}}^2 \pm \partial_x T_{\rm cl}$. It immediately follows that the mass squared $m_\pm^2$ is positive semidefinite, since

$$m_\pm^2 = \frac{\int dx\, \bar{\chi}_\pm\left(-\partial_x^2 + W(T_{\rm cl})^2 \pm \partial_x W(T_{\rm cl})\right)\chi_\pm}{\int dx\, \bar{\chi}_\pm\chi_\pm} = \frac{\int dx\ \left[(\partial_x \mp W)\bar{\chi}_\pm\right](\partial_x \mp W)\chi_\pm}{\int dx\, \bar{\chi}_\pm\chi_\pm} \geq 0. \tag{4.20}$$

Moreover, from this inequality we can show that there exist massless modes. The massless modes satisfy the first order equations

$$(\partial_x \mp W(T_{\rm cl}))\,\chi_\pm = 0. \tag{4.21}$$

This is easily solved as

$$\chi_\pm \propto \exp\left[\pm\int^x dx\, W(T_{\rm cl})\right]. \tag{4.22}$$

For the BSFT potential (2.2), we have $W(T_{\rm cl}) = T_{\rm cl}$, thus one can readily see that χ_+ is localized on the brane located at $x = x_1/2$, while the other massless mode for χ_- is on the antibrane at $x = -x_1/2$. The localized wave functions are almost gaussian, $\chi_\pm \sim \exp(-(x \mp x_1/2)^2)$.

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One can show that the mass spectra of χ_+ and χ_- are identical, except for the massless modes. Suppose that we have a solution for χ_+ in the eigen equation (4.18). Then constructing a new function

$$\widetilde{\chi}_- \equiv (-\partial_x + W(T_{\rm cl}))\chi_+\,, \tag{4.23}$$

it is easy to show that this $\widetilde{\chi}_-$ satisfies the χ_- eigen equation (4.18) with the mass m_+^2. This implies that for each eigen mode χ_+, there is an eigen state χ_- with the same mass, provided χ_+ is not a zero mode (then $\tilde{\chi} = 0$). The converse can be shown in the same manner, therefore we have degeneracy of the fermion spectra between χ_+ and χ_- except for the massless modes.

Since in our case the world volume is compactified, the wave functions are all normalizable. Hence both of the zero modes (4.22) are consistent solutions. Thus we conclude that the fermion spectrum contains one pair of zero modes and (infinitely) many pairs of massive modes. Namely fermions are doubly degenerate on our non-BPS brane-antibrane background even including massless modes.

Another way to see this degeneracy is as follows. For non-BPS configurations ($T_0 \neq \infty$) the solution we obtained has a period $2x_1$, and satisfies

$$T_{\rm cl}(x + x_1) = -T_{\rm cl}(x)\,. \tag{4.24}$$

Hence this translation along x gives an exchange $V^{(+)} \leftrightarrow V^{(-)}$. This provides an interesting relation between the eigen functions

$$\chi_\mp(x + x_1) = \chi_\pm(x)\,, \tag{4.25}$$

Therefore it is a direct result of this relation that the spectrum is doubly degenerate. This argument is valid even for the massless modes. However, for the BPS case (a single D-brane solution), there is no periodicity and thus the degeneracy cannot be observed in this way. In fact, the degeneracy is only for the massive modes in this BPS case [13]. One of the zero modes (4.22) becomes non-normalizable.

4.3 Gauge fluctuation

It is possible to analyse the gauge fluctuation in the same manner. The lagrangian we consider is

$$\mathcal{L} = -\left(\mathcal{K}'(T)\right)^2 \left(1 + \partial_\mu T \partial^\mu T + \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right). \tag{4.26}$$

For the two-derivative truncation of the BSFT, we have (2.2) [13, 15]. Performing the same analysis given in [12, 13] leads to the following Schrödinger equation

$$\left(-\partial_x^2 + V_{\rm gauge}(T_{\rm cl})\right) \hat{A}_{\hat{\mu}} = m_{\rm gauge}^2 \hat{A}_{\hat{\mu}}\,, \tag{4.27}$$

where $\hat{A}_{\hat{\mu}} \equiv \mathcal{K}'(T_{\rm cl}) A_{\hat{\mu}}$ and the potential is given by

$$V_{\rm gauge}(T_{\rm cl}) \equiv \left(\frac{\mathcal{K}''}{\mathcal{K}'}\right)^2 + \left[\frac{\mathcal{K}'''}{\mathcal{K}'} - \left(\frac{\mathcal{K}''}{\mathcal{K}'}\right)^2\right] (\partial_x T_{\rm cl})^2\,. \tag{4.28}$$

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The important point is that the operator in the Schrödinger equation can be recast into a form

$$-\partial_x^2 + V_{\rm gauge}(T_{\rm cl}) = D^\dagger D\,, \tag{4.29}$$

where the linear differential operator D and $D^\dagger$ are defined as

$$D \equiv i\left(\partial_x - \frac{\mathcal{K}''}{\mathcal{K}'}(\partial_x T_{\rm cl})\right) = i\mathcal{K}'\partial_x\frac{1}{\mathcal{K}'}\,, \qquad D^\dagger \equiv -i\left(\partial_x + \frac{\mathcal{K}''}{\mathcal{K}'}(\partial_x T_{\rm cl})\right) = i\frac{1}{\mathcal{K}'}\partial_x\mathcal{K}'\,. \tag{4.30}$$

These operators D and $D^\dagger$ are adjoint of each other with respect to the inner product for $\hat{A}_{\hat{\mu}}$ fields $(\phi_1(x), \phi_2(x)) \equiv \int dx \phi_1(x)\phi_2(x)$, as

$$(\phi_1(x), D\phi_2(x)) = (D^\dagger\phi_1(x), \phi_2(x))\,. \tag{4.31}$$

Defining the eigen function $u_n(x)$ as $D^\dagger D u_n = m_n^2 u_n$, we obtain from the expression (4.29) that the mass squared $m_{\rm gauge}^2$ is positive semi-definite

$$m_n^2(u_n, u_n) = (u_n, D^\dagger D u_n) = (Du_n, Du_n) \geq 0\,. \tag{4.32}$$

This implies that the massless mode should satisfy a linear differential equation

$$Du_0(x) = 0 \qquad \text{for} \qquad m_0 = 0\,. \tag{4.33}$$

From the last expression in (4.30), this massless mode can be easily found as

$$u_0(x) = \mathcal{K}'(T_{\rm cl}(x))\,. \tag{4.34}$$

Therefore we have found that the mass spectrum of the gauge fluctuations consists of one massless mode and massive modes, thus there is no tachyonic state.

4.4 Supersymmetry breaking

Although there are massless modes for all sectors of the fluctuation (tachyon (scalar), fermion and gauge fields), a tachyonic mode exists only in the tachyon (scalar) fluctuation t, as shown in (4.12). This is a reflection of the fact that the supersymmetry is broken for our solution. The amount of the supersymmetry breaking is measured by the difference of the boson/fermion spectra, that is (4.12) for the BSFT potential (2.2).

The massless mode of the tachyon fluctuation is the NG boson associated with the breaking of the translation invariance. Analogously, the massless modes of the fermion fluctuation can be thought of as NG fermions coming from the breaking of the overall target space supersymmetries as follows.

Non-BPS branes in superstring theory are unstable (meta-stable) and the associated tachyon T develops a vacuum expectation value. To describe this tachyon condensation, an effective field theory of tachyons away from $T = 0$ has been proposed. Since this field theory is originally a field theory on the non-BPS branes, the supersymmetries are completely broken at $T = 0$ and are realized only nonlinearly [15]. This is the target space supersymmetries of $1 + 9$ dimensions. In the limit of tachyon condensation $T \to$

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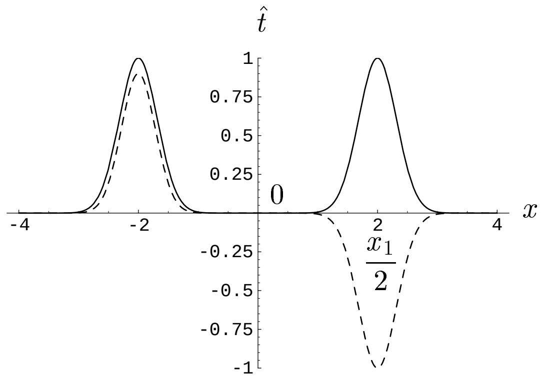

Figure 2: Two lowest fluctuations of the tachyon field T. The solid line denotes the tachyonic fluctuation, while the dashed line is the massless NG mode.

$\pm\infty$, supersymmetry is recovered. If we have a linear solution (2.4) of the effective field theory corresponding to a single BPS brane, half of the supersymmetries is recovered and is linearly realized. On the other hand, the brane-antibrane configuration breaks this recovered linearly supersymmetries. Therefore we should have the NG fermion associated with the broken target space supersymmetries.

Let us elaborate the above consideration more precisely. The total configuration of the brane and antibrane breaks supersymmetries completely. The amount of supersymmetry breaking is measured by the mass squared of the tachyon fluctuation (4.12). There is another way to observe this. The non-linearly realized supersymmetries of the original action with the fermions (the sum of (2.1) and (4.14)) are given as [15]

$$\delta\psi = \epsilon - (\partial_\mu T)\Gamma^\mu \epsilon\,. \tag{4.35}$$

Therefore one can see that, for the linear solution (2.4), half of the nonlinear supersymmetries are restored to be linear supersymmetries. This shows that the kink is a BPS D-brane. In our case, near the branes $x = \pm x_1/2$ the solution is approximated by a linear function $T \sim \mp q(x \pm x_1/2)$ (2.15), where $q \sim 1 + \mathcal{O}(\mathrm{e}^{-T_0^2})$. Plugging this into the above supersymmetry transformation, we find that it is slightly broken by $\mathrm{e}^{-T_0^2}$. This is consistent with the fact that in the fluctuation spectra the tachyon mass squared is of order $\mathcal{O}(\mathrm{e}^{-T_0^2})$, as shown in (4.12).

Another consistency check can be made through the low-energy theorem associated with the supersymmetry breaking [7, 16]. Since the linearly realized supersymmetry of the tachyon condensed vacuum ($T = \pm\infty$) is spontaneously broken by the brane-antibrane configuration, low-energy theorems apply for the coupling of the NG fermion. When supersymmetry is spontaneously broken, the supercurrent J^μ at vanishing momentum reduces to the NG fermion field $\psi_{\rm NG}$

$$J^\mu_\alpha = \sqrt{2}if\cdot(\gamma^\mu\psi_{\rm NG})_\alpha + J^\mu_{\mathrm{matter},\alpha} + \cdots\,, \tag{4.36}$$

where f is the order parameter of the supersymmetry breaking and $J^\mu_{\mathrm{matter},\alpha}(x)$ is the supercurrent for matter fields. The conservation of the supercurrent $\partial_\mu J^\mu_\alpha = 0$ relates the matrix elements of NG fermion to that of the matter supercurrent. If we evaluate them

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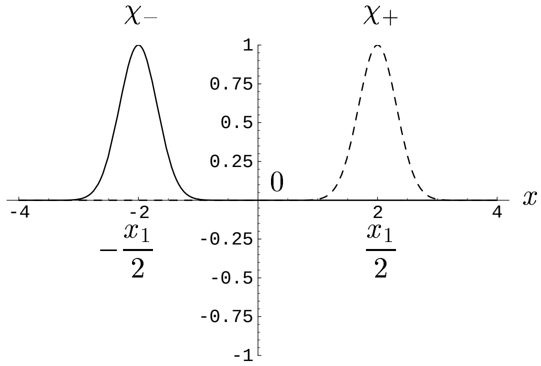

Figure 3: Fermionic fluctuation $\chi_\mp$ localized at $\mp x_1/2$ is shown by a solid (dashed) line.

between boson and fermion states of a supermultiplet, the matrix element of the NG fermion gives the effective three point coupling $g_{\rm eff}$ of the NG fermion with the supermultiplet of boson and fermion, whereas that of matter supercurrent gives the difference of squared mass $\Delta m^2 = m^2_{\rm boson} - m^2_{\rm fermion}$ between the boson and the fermion. The resulting low-energy theorem is [16]

$$g_{\rm eff} = \frac{\Delta m^2}{f}\,. \tag{4.37}$$

The order parameter of the supersymmetry breaking f is given by the energy V_0 of the background solution as $f = \sqrt{V_0}$. The effective three point coupling can be given by the overlap integral of the three wave functions in x. If we take the massless fermion localized on the brane as the matter fermion, its superpartner is given by the boson localized on our brane which is approximately given by an equal weight superposition of the zero mode and the tachyonic mode as shown in figure 2. On the other hand, the NG fermion is localized on the antibrane as shown in figure 3. The three point coupling is given in terms of an overlap integral of the wave functions of the supermultiplet of the boson and fermion with the wave function of the NG fermion which can be efficiently evaluated by the single kink approximation [7]. Since the wave function of the NG fermion is localized at the antibrane and has an exponentially suppressed tail around the brane, the overlap integral is exponentially suppressed to give the order $\mathcal{O}(\mathrm{e}^{-T_0^2})$ as a function of T_0 in agreement with our result of the direct evaluation of the tachyon mass squared in (4.12).

It should be noted that the potentials for each fluctuation, $V_{\rm tachyon}$, $V^{(\mp)}_{\rm fermion}$, and $V_{\rm gauge}$, coincide with each other if the classical solution is that of the single brane, $T_{\rm cl} = \pm(x - x_0)$. Therefore in the BPS limit our result reproduces that of [13], however we note that in our computation the derivation of the potential does not refer to any particular expression of the solutions.

In sum, if the solution deviates from the linear case ($T_{\rm cl} = x$), the potentials for each fluctuation differ, and generically the degeneracy of the boson/fermion/gauge fluctuation spectra is resolved. For any classical background solution one finds as fluctuations a massless bosonic scalar mode (NG boson), two fermionic modes which are NG fermions, and a massless vector mode.[5]

[5]This massless vector mode is supposed to be confined after the annihilation of the brane and the antibrane (see for example [17, 3]).

5. Brane-antibrane annihilation

Since there is a tachyonic fluctuation, the obtained solution is meta-stable and decays by some quantum effect. In this section we describe how the constructed solution of the brane and antibrane decays. The most natural decay may be by approach of the brane and antibrane. As we noted, the vacuum expectation value (vev) of the tachyonic fluctuation corresponds to the distance between the brane and antibrane. Thus this decay mode is actually the rolling down of the potential hill of the tachyonic fluctuation mode. From the mass squared of it (4.12), one can write the potential around the vacuum as

$$V(t) \sim \frac{-T_0 e^{-T_0^2}}{2} t^2 + \mathcal{O}(t^4) . \tag{5.1}$$

The rolling down of the potential hill starts from quantum fluctuation, and a quantum-field-theoretical treatment for this kind of decay has been known for years [18, 19].[6] Quantum one-loop corrections to the potential gives an imaginary part, and that is regarded as a decay width of the system. In our case, the world volume theory is $p+1$ dimensional, then the decay width for a unit worldvolume can be obtained by following the computation in [19, 22] as

$$\Gamma = \frac{\pi}{((p+1)/2)!} \left(\frac{-m^2}{4\pi}\right)^{(p+1)/2} . \tag{5.2}$$

Here we derived this for the case of odd p for simplicity. m^2 is the tachyon mass which is now given by (4.12), therefore the rough estimate of the decay width is

$$\Gamma \sim \exp\left(-\frac{p+1}{2} T_0^2\right) . \tag{5.3}$$

This decay width describes the approaching of the branes to each other, as shown in figure 4. The size of the supporting domain of the deformed surface can be also estimated [19] as

$$r_0 \sim \frac{1}{m} \sim \exp(T_0^2) . \tag{5.4}$$

For the brane and antibrane, there is another interesting decay mode. This is nucleation of a throat connecting the two branes [24, 25]. The decay starts with a quantum tunnel effect by the nucleation of the throat, and then the throat expands to sweep out the brane and antibrane. The decay width can be easily computed by use of the tension of the branes and the brane separation. The radius of the nucleated throat is given as [25]

$$r = \frac{p}{2} x_1 , \tag{5.5}$$

where we used that $x_1 \sim 2T_0$ is the brane separation. Since the decay width is given by the action of the throat (which equals the tension times the area of the throat),

$$\Gamma \sim \exp\left(-T_0^{p+1}\right) . \tag{5.6}$$

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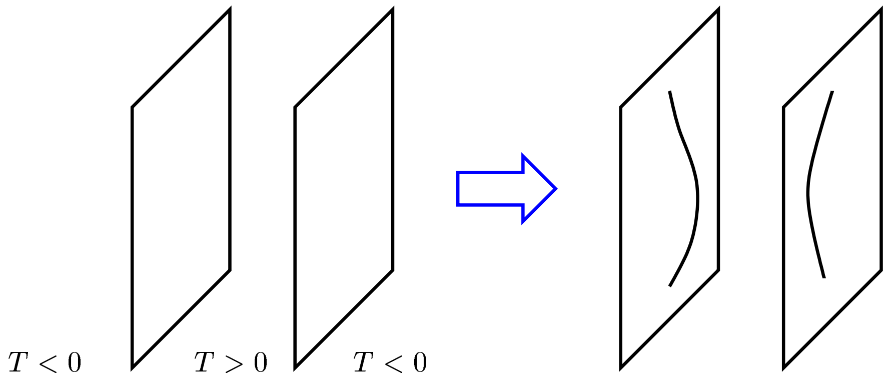

Figure 4: The surfaces of the brane and antibrane are deformed, and they start approaching.

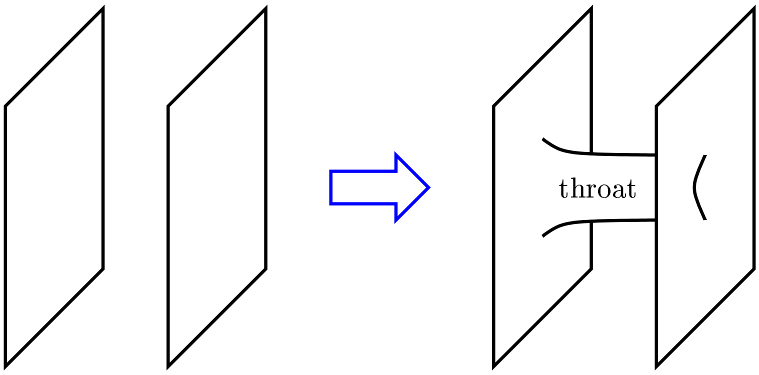

Figure 5: Nucleation of a throat connecting the brane and the antibrane.

Therefore this is not a dominant decay mode if $p > 1$. When $p = 1$, this decay process is comparable with the first one (gradual approaching).

For both decay modes, after the annihilation of the brane and antibrane, we will be left with an almost homogeneous tachyon vev $T \sim T_0$. This is unstable and rolls down the original potential e^{-T^2}, and finally we will be led to the rolling tachyon phase [26].

6. Conclusion and discussion

In this note we have constructed a classical solution representing a brane-antibrane pair in a compact spacetime, in the tachyon effective field theory known as Minahan-Zwiebach model. The force acting between the branes comes from the modes propagating between the brane and the antibrane. The mass of this mode is shown to agree with the excitation of the string stretched between the branes. Fluctuation spectra for the tachyon field, the fermions and the gauge fields are obtained, and they are found to exhibit the supersymmetry breaking structure. There are two possible decay modes of this configuration, one is the approach of the branes to each other, and the other is the nucleation of a throat connecting the branes. It is shown that the former decay mode is dominant in our theory.

[6] Application for the string theory context, see [20]. Discussion in the modern D-brane context, see [21]–[23].

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Although the tachyon effective theory considered here has many desirable properties, it may receive α' corrections in string theory, and there is no guarantee that the solution considered here survives. In fact, even for the linear tachyon profile of a single D-brane solution, the corresponding BSFT action includes infinitely many higher terms. A discussion on this higher derivative treatment is briefly given in appendix A. However, we point out here that our solution is equipped with a property which coincides with the solution of full string theory — a conformal field theory construction of brane and antibrane given by A.Sen [2]. There exists an exactly marginal deformation of the CFT which is a form of $\lambda \sin(X)$ where X is a scalar field in two dimensional worldsheet theory [27], and λ is a deformation parameter. This deformation is linear around $X \sim 0$ while $\sim X^2$ around the turning point (in our solution, around T_0). Since this deformation is exactly marginal, the configuration is a solution of full string theory at the tree level. Therefore an optimistic view is that our solution is not far from the full solution. It is important to construct an off-shell formalism around this marginally deformed background.[7]

The brane-antibrane interaction in string theory should be dictated by the exchange of gravitons and RR gauge fields. However in our setup we neglected these closed string modes from the first place, thus we cannot see this in the force estimation in section 3. In one sense this was good for us since we wanted to see the open string off-diagonal mode. However for the precise description of the brane and antibrane the lack of gravitational interaction is fatal, and how the closed string effect is incorporated in the tachyon model should be studied, for the precise description of the decay for example.

Our brane-antibrane solution can be formed dynamically, if we start from an unstable homogeneous vacuum $T = 0$. If the tachyon starts rolling inhomogeneously, generically the direction of the rolling may differ in regions of the world volume, and a stack of brane-antibrane pairs may be formed. The time-dependent inhomogeneous decay of tachyon theory was recently studied by several people [30, 25] in various approaches. Here again the inclusion of the closed string modes may be concern. We leave these issues to the future work.

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A. BSFT treatment with higher derivative terms

In the whole analysis in this paper, so far we have used the Minahan-Zwiebach model (2.1). Although this model is field theoretically very interesting and shares common properties

[7] See [28] for the related study in the context of tachyon effective theories. In Berkovits' formulation of superstring field theory, similar classical solutions were constructed in [29].

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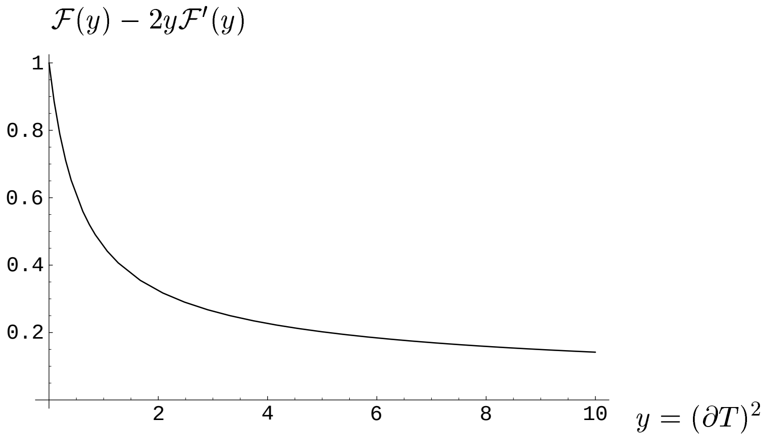

Figure 6: The right hand side of the equation (A.4).

with string theory low energy behavior, from a strict point of view, it is just a derivative truncation of the BSFT action [10, 11] and receives higher derivative corrections. In this section we demonstrate to what extent the examined properties of the brane-antibrane solution survives in the derivative corrections.

The BSFT action for the linear tachyon profile (equivalently, neglecting the $\partial\partial T$ terms) is given in [11] as

$$S = -\int d^{p+2}x \mathrm{e}^{-T^2/4} \mathcal{F}(y) \,, \tag{A.1}$$

where

$$\mathcal{F}(y) = \frac{y 4^y (\Gamma(y))^2}{2\Gamma(2y)} \,, \qquad y \equiv (\partial_\mu T)^2 \,. \tag{A.2}$$

We can solve the equations of motion in a similar way, by constructing a conserved quantity for the translational invariance,

$$A = \frac{\delta L}{\delta \partial T} \partial T - L = \mathrm{e}^{-T^2/4} \left(\mathcal{F} - 2y\mathcal{F}' \right) . \tag{A.3}$$

Therefore, the redefinition of the constant $A = \mathrm{e}^{-T_0^2/4}$ leads to

$$\mathrm{e}^{(T^2 - T_0^2)/4} = \mathcal{F} - 2y\mathcal{F}' \,. \tag{A.4}$$

The MZ model has $\mathcal{F}(y) = 1 + y$. Therefore the right hand side of the above equation is linear, $\mathcal{F} - 2y\mathcal{F}' = 1 - y$. It becomes zero at $y = 1$, and this indicates that the solution at $T = 0$ for large $|T_0|$ becomes almost linear with slope $|\partial T| = 1$. On the other hand, in the BSFT case, the right hand side has no zero (see figure 6). The limit $\mathcal{F} - 2y\mathcal{F}' \to 0$ corresponds to $y \to \infty$. This means that, the slope q at which $T = 0$ (the location of the branes) is very large. This slope q diverges if we take the BPS limit $T_0 = \infty$, which is consistent with the previous analyses in the BSFT.

As before, the estimation of the slope q may lead to the evaluation of the brane-antibrane separation. In the asymptotic region of y we can approximate the right hand side of (A.4) as

$$\mathcal{F} - 2y\mathcal{F} \sim \sqrt{\frac{\pi}{16y}} + \mathcal{O}\left(\frac{1}{y^{3/2}} \right) . \tag{A.5}$$

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Therefore for large $|T_0|$, the slope at $T = 0$ can be approximated by the solution

$$-\frac{1}{4}T_0^2 \sim -\frac{1}{2}\log y\,. \tag{A.6}$$

Since in this linear approximation the inter-brane separation x_1 is given by $x_1 = T_0/q$, we substitute $y(T = 0) = q^2 = (T_0/x_1)^2$ into the above equation and obtain

$$x_1 \sim |T_0|\mathrm{e}^{-T_0^2/4}\,. \tag{A.7}$$

Unfortunately, this means that the large $|T_0|$ does not corresponds to the large separation. Hence our linear approximation is invalid. However, it is obvious from figure 6 that a similar oscillatory solution exists and exhibits periodicity. Since we have not found a suitable limit of the parameters for the large brane separation, it seems to be difficult to see the structure of the supersymmetry breaking without performing explicit fluctuation analysis using the solutions, although it is obvious that the supersymmetry breaking occurs. We expect similar processes for the brane-antibrane annihilation.

Though here we have considered only the higher derivative corrections of the form $(\partial T)^n$ $(n \geq 4)$, inclusion of the terms with higher derivatives acting on T, such as $\partial^2 T$, would be also important. The relevant action was analysed in [31], and it would be interesting by using the action in [31] to see if the brane-antibrane solution survives also these corrections.[8]

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